

The Minimal Spanning Tree Algorithm

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Introduction

The minimum-weight spanning tree problem (MSTP), one of the most typical and well-known problems of combinatorial optimization, is that of finding a spanning tree of an undirected, connected graph, such that the sum of the weights of the selected edges is minimum. My main -objective Minimum Spanning Tree (MST) model has been used successfully for finding optimal solutions in network problems. For example, it has been shown by numerical simulation that in transportation and communication infrastructures the optimal network is a spanning tree [4] ; and it is suggested that transport in weighted networks is dominated by the MST [7]. Moreover, the hierarchical network design problem has been solved by calculating the MST of the corresponding graph [3]. An instance of the minimum spanning tree problem consists of an undirected graph $G = (V, E)$ with n vertices and m edges and a positive integer weight $w(e)$ for each edge. The problem is to find an edge set E connecting all vertices of V with minimal total weight.

Methodology

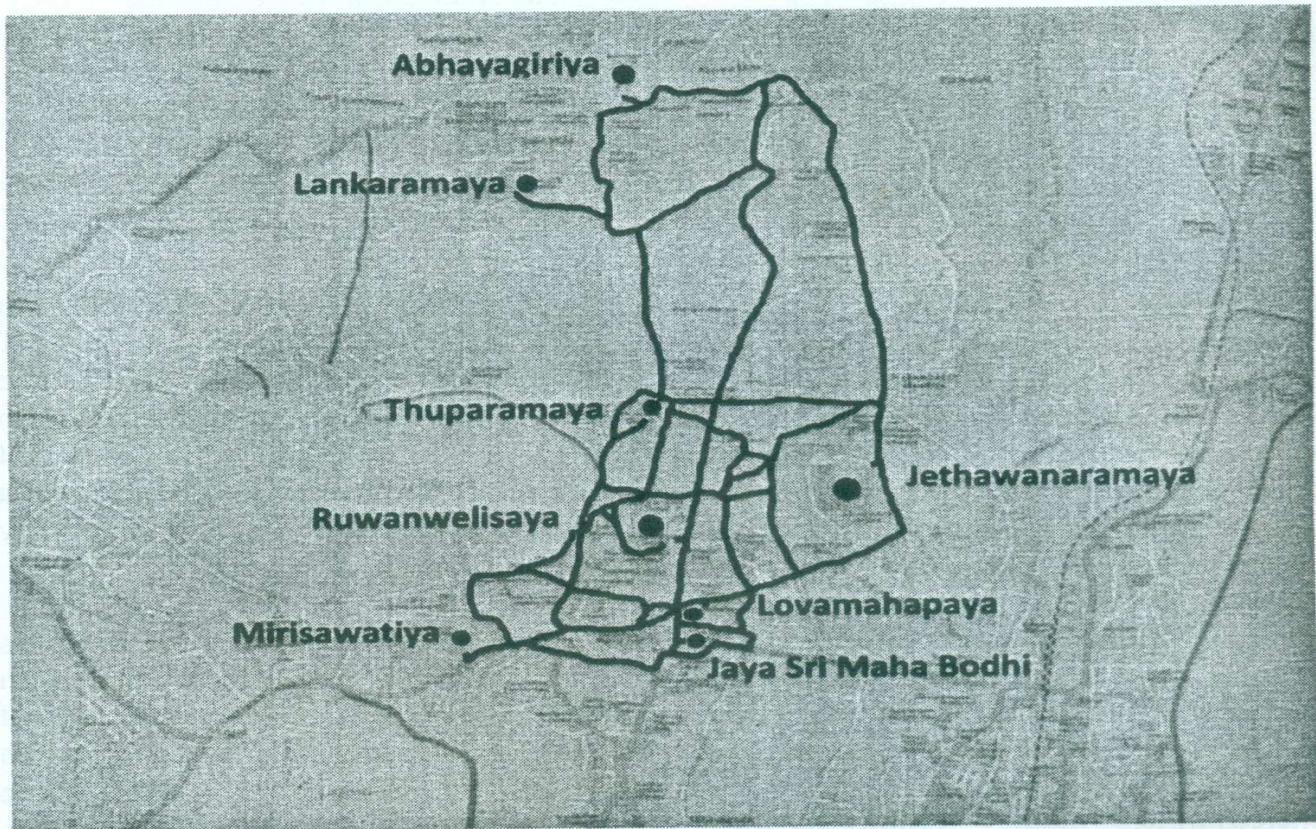
2.1 Definition (Spanning Trees)

A spanning tree $T = (V, E)$ is a connected subgraph of G which contains

no cycle and all vertices of G . (Note that original graph G and the spanning tree T have the same set of vertices.) The problem is to find a spanning tree with minimal costs under the constraint that the weight of the tree is not greater than a given constant W . I can state my problem in the following way.

The minimal spanning tree problem is one of the most popular problems in graph theory and therefore a lot of literature concerning fast algorithms can be found. In this paper , collect some basic definitions from graph theory, properties concerning spanning trees and some facts for the minimal spanning tree problem which might become useful for our problem. Also I use the two optimality conditions for the minimal spanning tree problem to obtain optimality conditions for the weight-constrained minimal spanning tree problem. The problem is to determine how to connect Eight sacred places in thenetwork using the minimal distance .Atamasthana (අටමස්ථානය) or Eight sacred places are a series of locations in Sri Lanka where the Buddha had visited during his three visits to the country. The sacred places are known as Jaya Sri MahaBodhiya, Ruwanwelisaya, Thuparamaya, Lovamahapaya, AbhayagiriDagaba, Jetavanarama,

MirisavetiStupa and Lankarama. They are situated in Anuradhapura. We can model this problem by drawing a network.



$G=(V,E)$

2.2 The steps of the procedure are given as follows;

Let $N=\{1,2,3,\dots,n\}$ be the set of nodes of the network and define ,

C_k = Set of nodes that have been permanently connected at iteration k of the algorithm

$\overline{C_k}$ = Set of nodes as yet to be connected permanently.

Step 0: Set $C_0 = \Phi$ and $\overline{C_0} = N$

Step 1: Start with any node i , in the unconnected set $\overline{C_0}$ and set $C_1 = \{i\}$, which automatically renders and $\overline{C_1} = N - i$

General step K :

Select node j , in the unconnected set $\overline{C_{K-1}}$ = that yields the shortest branch to a node in the connected set C_{K-1} .

Link j permanently to C_{K-1} and remove it from $\overline{C_{K-1}}$, that is

$$C_k = C_{K-1} + [j] , \quad \overline{C_k} = \overline{C_{K-1}} - [j]$$

If the set of unconnected nodes , $\overline{C_k}$ is empty,. Now stop. Otherwise set $k = k+1$ and repeat the step.

1) **Experimental results**

The main research outcome of this project is to determine the shortest routes between the origin and the destination nodes in the network. In my example, determined the best route from Anuradapura to each of the eight destination Temples (Atamasthana). The minimal spanning tree problem is similar to the shortest route problem, except that the objective is to connect all the nodes in the network so that the total branch lengths are minimized. The resulting network connects all the points in the network at a minimum total distance (or length).

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